

## Wakefield acceleration in planetary atmospheres: A possible source of MeV electrons. The collisionless case

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### ABSTRACT

Intense electromagnetic pulses interacting with a plasma can create a wake of plasma oscillations. Electrons trapped in such oscillations can be accelerated under certain conditions to very high energies. We study the optimal conditions for the wakefield acceleration to produce MeV electrons in planetary plasmas under collisionless conditions. The conditions for the optimal plasma densities can be found in the Earth atmosphere at higher altitudes than 10–15 km, which are the altitudes where lightning leaders can take place.

### 1. Introduction

Electromagnetic pulses interacting with a plasma can create a wake of plasma oscillations through the action of the nonlinear ponderomotive force (Tajima and Dawson, 1979). Electrons trapped in the wake can be accelerated to high energies, achieving GeV/m accelerating gradients. In the laboratory very high-power electromagnetic radiation from lasers is used to accelerate electrons to high energies in a short distance. However, in atmospheric plasmas the distance range goes from metres to kilometres, and the conditions for wakefield acceleration to take place are different as the plasma characteristics are different. We will study in this work the interaction of an electromagnetic wave with a plasma under collisionless conditions in atmospheric plasmas.

The acceleration of the electrons in atmospheres has been the subject of attention since the discovered of Terrestrial Gamma Flashes (TGF's). TGF's were first discovered by accident in 1994 (Fishman et al., 1994) and only limited experimental data is available (Smith et al., ). One of the goals of the European Space Agency (ESA) mission ASIM (ASIM, ) will be to provide experimental data on TGF's. Any theory about the TGF generation should be able to successfully explain the observed duration, energy spectra, and photon fluence (Tavaniet al., 2011). There is some agreement that the emission is produced via Bremsstrahlung (Shao et al., ) when high energy electrons collide with nuclei in the air releasing energy. The satisfactory theory for TGF's needs to explain the origin of the high-energy electrons (Gurevich et al., 1992) which can trigger the

ignition of the gamma flashes (Smith et al., 2005). At present there are two realistic TGF's mechanism. One is called the Relativistic Runaway Electron Avalanche (RREA) feedback mechanism, and for a detailed description we point to the reference (Dwyer et al., 2012). The other is the leader/streamer theory (Celestin and Pasko, ), in which seed electrons are produced in the streamer head and accelerated in the stepped leader electric field (there is the current pulse version of that (Carlson et al., )).

The wakefield phenomena could also supply the initial seed of energetic electrons, coming from the interaction of an electromagnetic pulse with an existing plasma previously created. Although the idea of production of high energy electrons by an electromagnetic pulse due to a lightning return stroke was explored previously (Inan and Lehtinen, ), the novelty of our proposal is the interaction of the pulse with a plasma already present in the atmosphere and created for example by another discharge or any electromagnetic activity.

In order to explore the idea, we assume that the acceleration takes place within the plasma under collisionless conditions, and thus the friction force (Bethe, 1930) due to collisions with neutral molecules, which is the main braking force in other theories (Gurevich and Zybin, 2001) is not considered. That is valid provided that some conditions for the pulse and plasma are fulfilled. We establish those conditions in the present study. Whether the inclusion of electron-neutral collisions is necessary, the conditions should be modified. That would be the subject of future work.

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The outline of the paper is the following. First we summarize the optimal conditions under which an electromagnetic wave can create a wakefield in a plasma, so acceleration of electrons to MeV energies is possible. The acceleration mechanism is demonstrated through computer simulations and we establish a criteria for this to work. We use a unipolar and bipolar form of the electromagnetic pulse as found in lightning for example (Ka spar et al., 2015). We provide some discussions for the production of the plasma and the electromagnetic pulse in planetary atmospheres, and in particular for the Earth atmosphere. Finally we summarize the main results and end with some conclusions.

## 2. Optimal conditions for wakefield acceleration

When an electromagnetic wave encounters a plasma it can create a wake inside the plasma through the action of a nonlinear ponderomotive force (Tajima and Dawson, 1979). The following is a set of optimal conditions (derived from a single-particle approximation discussed in detail in Appendix A), required to accelerate electrons to MeV energies.

First we need a plasma region with the following characteristics. The numbers of particles in the Debye sphere must be large,  $N_D \gg 1$ . For electrons,  $N_D = 4\pi n_e \lambda_{De}^3 / 3$ , where  $n_e$  is the electron density and  $\lambda_{De}$  the electron Debye length. A useful expression of the Debye length under equilibrium conditions is given by  $\lambda_{De} = 69 \sqrt{T/n_e}$  (m), where  $T$  is the equilibrium temperature of the electrons in Kelvin and  $n_e$  is expressed in  $m^{-3}$  (Spitzer, 1962), so we have

$$N_D \approx 1.38 \times 10^6 T^{3/2} / n_e^{1/2} \gg 1. \quad (1)$$

The Debye length must be much smaller than the characteristic size of the plasma region,

$$\lambda_{De} \ll L. \quad (2)$$

We recall that the condition to have a plasma is  $\lambda_{De}$  being small than the characteristic dimension of the system.

Now the coupling conditions for the electromagnetic wave and the plasma. The frequency of electron and ions collisions must be smaller than the electromagnetic pulse angular frequency,

$$\nu_{ei} \ll \omega_0. \quad (3)$$

The damping rate also must be negligible,

$$\nu = \frac{\omega_{pe}^2}{\omega_0^2} \nu_{ei} \ll \omega_{pe}, \quad (4)$$

where  $\omega_{pe} = e\sqrt{n_e/(m_e \epsilon_0)}$ . The physical meaning of this condition comes as  $\nu$  represents the rate of energy lost from the electromagnetic wave, i. e.,  $\nu(E_0^2 \epsilon_0)/2$ . It must be balanced by the rate at which the oscillatory energy of the electrons  $n_e e^2 E_0^2 / 2m_e$  is dissipated by electron-ion scattering with frequency  $\nu_{ei}$ . If we assume a wave package with group velocity  $v_g$ , the energy of the wave will decay within a characteristic length of  $l = v_g/\nu$ , and we should expect that to be bigger or at least the order of the Debye length,  $\lambda_{De} \leq l$ .

For the acceleration of electrons to MeV energies, we need the further condition derived from our simulations,

$$\tilde{\omega} = \frac{m_e c \omega_0}{e E_0} < 1. \quad (5)$$

We will discuss further this condition in the next section.

## 3. The acceleration of electrons in atmospheric plasmas

In this section we will study how electrons in a plasma are accelerated by an electromagnetic pulse propagating through the plasma under the collisionless conditions. We will first set the equations to describe the electron dynamics and then compare their predictions with some particle

in cell code simulations to verify the results. We will use in particular two kind of pulses, a unipolar and a bipolar one, as they are a good approximation for the pulses that one can find for example in the electromagnetic activity of the Earth atmosphere (Eack, ; Risonet al, ).

Let us consider an electromagnetic pulse which is propagating in the x-direction inside a plasma,

$$\mathbf{E}(\mathbf{r}, t) = \int dk A(k) e^{i(kx - \omega t)} \hat{\mathbf{e}}_y + c.c., \quad (6)$$

where  $A(-k) = A(k)^*$  and  $A(k)$  for  $k > 0$  is nonzero only in the vicinity of a central wave number  $k_0$ . From the relation (A.14) we take for the pulse a lead frequency  $\omega_0 = (w_{pe}^2 + c^2 k_0^2)^{1/2}$ . Expanding around  $k_0$  yields the following expressions for the electromagnetic field,

$$\begin{aligned} \mathbf{E}(\mathbf{r}, t) &= E_0 f(x - v_g t) \cos[(\omega_0 - v_g k_0)t] \hat{\mathbf{e}}_y, \\ \mathbf{B}(\mathbf{r}, t) &= \frac{v_g}{c^2} E_0 f(x - v_g t) \cos[(\omega_0 - v_g k_0)t] \hat{\mathbf{e}}_z, \end{aligned} \quad (7)$$

where

$$v_g = \left( \frac{\partial \omega}{\partial k} \right)_{k=k_0} = \frac{c^2 k_0}{\omega_0} = c \epsilon^{1/2} \quad (8)$$

is the propagation speed inside the plasma,  $\epsilon$  is the plasma dielectric function (A.11), and  $f(x) = E_y(\mathbf{r}, 0)/E_0$  gives the shape of the pulse,  $E_0$  being the amplitude of the electric field.

As discussed in the previous section, the condition  $\omega_0 \geq \omega_{pe}$  is required for the propagation of the electromagnetic pulse inside the plasma. In the following we will consider  $\omega_0 \geq 10 \omega_{pe}$ . For  $\omega_0 = 10 \omega_{pe}$ ,  $v_g$  differs from the speed of light in vacuum in less than 1%. For larger values of  $\omega_0$ , the pulse propagation speed is even closer to  $c$ . The electromagnetic pulse will start interacting with the electrons at  $t = 0$ . The interaction will end at a time of the order of  $2\pi/\omega_0$ . Since  $\omega_0 - v_g k_0 = \omega_{pe}^2/\omega_0$ , the cosine term in (7),  $\cos[(\omega_0 - v_g k_0)t] = \cos[(\omega_{pe}/\omega_0)^2 \omega_0 t] \approx 1$  can be safely neglected throughout the process. We assume that the damping rate (A.15) of the pulse is negligible. These assumptions will impose some constrains that we will discuss at the end of this section. Using these approximations, the electromagnetic field can be written as

$$\mathbf{E}(\mathbf{r}, t) = E_0 f(x - ct) \hat{\mathbf{e}}_y, \quad (9)$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{E_0}{c} f(x - ct) \hat{\mathbf{e}}_z. \quad (10)$$

Since the pulse frequency is larger than the plasma frequency, and the plasma frequency bigger than the collision frequency, the dynamics of the electrons will be collisionless while interacting with the pulse, so can be modelled by (Landau and Lifshitz, 1975)

$$\frac{d\mathbf{v}}{dt} = -\frac{e}{m_e} \left( 1 - \frac{v^2}{c^2} \right)^{1/2} \left[ \mathbf{E} + \mathbf{v} \times \mathbf{B} - \frac{\mathbf{v}(\mathbf{v} \cdot \mathbf{E})}{c^2} \right]. \quad (11)$$

We define dimensionless variables  $\tilde{t} = tE_0/(m_e c)$  and  $\tilde{\mathbf{r}} = \mathbf{r}E_0/(m_e c^2)$ , so that the scaled velocity is just  $\tilde{\mathbf{v}} = \mathbf{v}/c$ . Using (9) and (10), the equation of motion (11) becomes

$$\begin{aligned} \frac{d\tilde{v}_x}{d\tilde{t}} &= -\sqrt{1 - \tilde{v}^2} (\tilde{v}_y - \tilde{v}_x \tilde{v}_y) \tilde{f}[\tilde{\omega}(\tilde{x} - \tilde{t})], \\ \frac{d\tilde{v}_y}{d\tilde{t}} &= -\sqrt{1 - \tilde{v}^2} (1 - \tilde{v}_y^2 - \tilde{v}_x) \tilde{f}[\tilde{\omega}(\tilde{x} - \tilde{t})], \\ \frac{d\tilde{v}_z}{d\tilde{t}} &= -\sqrt{1 - \tilde{v}^2} (-\tilde{v}_z \tilde{v}_y) \tilde{f}[\tilde{\omega}(\tilde{x} - \tilde{t})], \end{aligned} \quad (12)$$

where  $\tilde{f}(k_0 x) = f(x)$  takes into account the assumed form of the wave-packet, and

$$\tilde{\omega} = \frac{m_e c \omega_0}{e E_0} \quad (13)$$

We are interested in pulses which are able to accelerate electrons to kinetic energies  $K = m_e c^2 [(1 - \tilde{v}^2)^{-1/2} - 1]$  in the order of several MeV after the interaction time, which is  $\tilde{T} \approx 2\pi/\tilde{\omega}$ . Therefore, in this model the interaction with the pulse is controlled by the dimensionless parameter  $\tilde{\omega}$ . Pulses with different amplitudes and frequencies but same ratio  $\omega_0/E_0$  (and shape) produce the same acceleration.

Fig. 1 shows the relativistic kinetic energy obtained for a free electron initially at rest at the origin accelerated by a pulse with the shape of the form

$$\tilde{f}(\tilde{\omega}x) = \cos[\tilde{\omega}(x - \tilde{T}/4)] [H(x + \tilde{T}/2) - H(x)], \quad (14)$$

which is unipolar (see right of Fig. 1), and also for the bipolar shape

$$\tilde{f}(\tilde{\omega}x) = \cos[\tilde{\omega}(x - \tilde{T}/4)] [H(x + \tilde{T}) - H(x)], \quad (15)$$

where  $H(x)$  is the Heaviside step function. The results were obtained by solving numerically (12). The simulations show that for values  $\tilde{\omega} \lesssim 0.1$ , regardless whether the shape is unipolar or bipolar, the kinetic energy keeps growing after five periods  $T_0$ , which is an indication that the electron is actually trapped by the pulse, soon achieving very high energies. For  $\tilde{\omega} \approx 1$ , the electromagnetic pulse is not so strong as to carry away the electron. It is still able to produce accelerations to energies in the MeV scale, though for the bipolar pulse the electron acceleration is later reversed by the own electromagnetic packet. For larger values of the reduced frequency,  $\tilde{\omega} \gtrsim 10$ , the impulse produced by the pulse is well below the MeV range.

These results were numerically verified using the Finite Difference Time Domain Particle in Cell software VORPAL (Nieter and Cary, 2004). Fig. 2 shows the kinetic energy profile after a time interval of  $1.44 \times 10^{-7}$  s, for a plasma of density  $n_0 = 10^{10} \text{ m}^{-3}$  with a unipolar pulse defined by the boundary condition  $E(0, y, z, t) = E_0 \sin(\omega_0 t) H(\pi/\omega_0 - t) \hat{e}_y$ , where  $H$  is the Heaviside step function. For  $E_0 = 10^5 \text{ V/m}$  and  $\tilde{\omega} = 1.071$ , as predicted, the pulse is able to produce electrons just in the MeV scale, whereas the pulse with  $E_0 = 10^6 \text{ V/m}$  yields  $\tilde{\omega} = 0.1071$ , able to accelerate free electrons to very large energies. Note that for the case of the higher electric field amplitude, the acceleration occurs at larger distances.

In Fig. 3, we have plotted the threshold conditions for which the wakefield is able to accelerate electrons to MeV energies, based on the requirements imposed by condition (5). A yellow a band around 50–60 MHz, which corresponds to the typical emitted frequencies in negative leaders, has also been plotted in Fig. 3 for reference. A range of pulse intensities where  $\tilde{\omega}$  is less than unity can be observed, so in principle the acceleration to MeV energies is possible under these conditions. It can be observed that as the peak amplitude increases from the lower

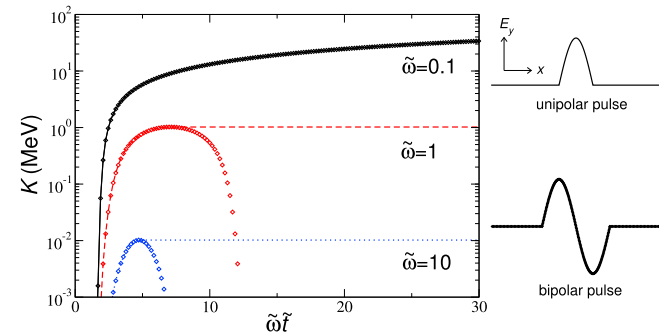


Fig. 1. Kinetic energy vs time of an electron accelerated by a unipolar (dotted lines) and a bipolar (crosses) EM pulse, for several  $\tilde{\omega}$ . The pulses, depicted in the right side, are assumed to be translation invariant in the directions perpendicular to the direction of propagation (along the  $x$ -axis).

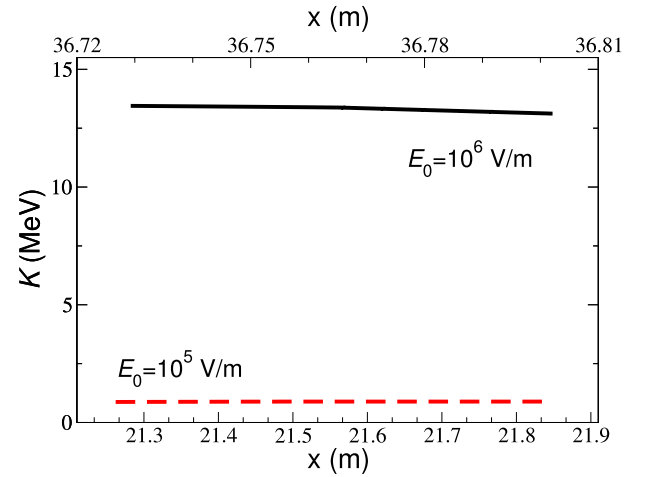


Fig. 2. Energy of electrons in a plasma of density  $n_0 = 10^{10} \text{ m}^{-3}$  after an acceleration time interval of  $1.44 \times 10^{-7}$  s due to an electromagnetic unipolar pulse of angular frequency  $\omega_0 = 2\pi \times 10^7$  rad/s. The continuous line corresponds to a pulse of amplitude  $E_0 = 10^6 \text{ V/m}$  and its abscissa is the upper one. The dashed line correspond to  $E_0 = 10^5 \text{ V/m}$ , being its abscissa the lower one. The plasma frequency is  $\omega_{pe} = 5.6 \times 10^6$  rad/s.

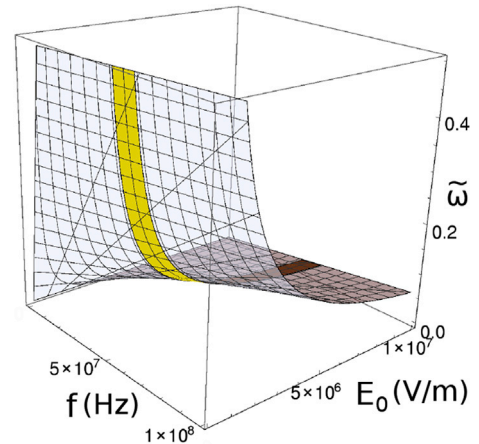


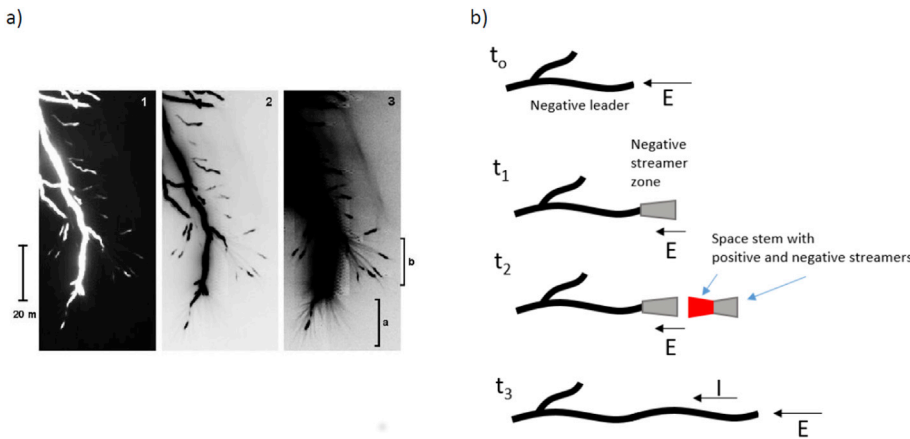
Fig. 3. The phase diagram for wakefield conditions able to accelerate electrons to MeV energies. The yellow a band around 50–60 MHz indicates the typical emitted frequencies in negative leaders. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

value  $5 \times 10^5 \text{ V/m}$ , the parameter  $\tilde{\omega}$  decreases, so the acceleration is more effective.

Still, the rest of conditions (1)–(4) must be fulfilled. For instance, choosing —as in the above simulations—  $\omega_{pe} = \omega_0/10$ , condition (4) is satisfied provided that the electron-ion collision rate is not larger than a magnitude of 100. This imposes some restrictions for the plasma in which the EM pulse propagates. The plasma density is related to the plasma frequency so for the 50 – 60 MHz band, the plasma density should be of the order  $10^{10} \text{ m}^{-3}$ . If we assume a plasma temperature of a few eV, and the mean atomic number of the air  $Z = 7.2$  (Earth atmosphere), it can be easily checked that conditions (1) and (2) are satisfied. With those values, the characteristic plasma length  $\lambda_{De}$  is tens of centimeters, and  $\nu_{ei}$  is less than 1 rad/s, which is much less than  $\omega_0$ , in agreement with (3).

### 3.1. MeV electron production in the Earth atmosphere

In this section we will apply the theory developed in previous sections in order to discuss few scenarios in the Earth atmosphere. We have shown



**Fig. 4.** Video images and sketch of the stepping process of negative leaders. a) Standard video frame and inverted images of a negative downward leader (adapted from (Petersen and Beasley, 2013)). b) Sketch of the stepping process of a negative leader. At time  $t = t_0$ , very intense electric fields are found at the leader tip, which will be quickly screened by the formation of the streamer zone at  $t = t_1$ . At the instant  $t_2$ , a space stem is formed with a bidirectional development that will finish with the encounter of the negative and positive streamers, leading a high current impulse at  $t_3$ .

previously how an electromagnetic pulse, interacting with an already created plasma, is able to accelerate electrons to high MeV energies provided some conditions are fulfilled. In a laboratory, the electromagnetic pulse is provided by a laser source, and the plasma is created ionizing a gas at certain pressure, either using microwaves or any other technique. In a planetary atmosphere, the pulse must be created by electromagnetic activity, such as lightning, leader strokes, etc. The plasma also must be present, and the again some electromagnetic activity is needed for the creation of the plasma.

Candidates in the Earth atmosphere for the creation of the pulse could be for example lightning discharges cloud to cloud, or cloud to earth (Eack, ; Risonet al, ), lightning leader, blue jets, etc. When the pulse is created, it propagates at basically the speed of light through the planetary atmosphere. If that pulse finds in its way a plasma, then the wakefield mechanism might be fired. Also those phenomena can be the source of several plasma regimes. For example intense and pulsating electric fields can be found in lightning leaders (Fig. 4a) and leaders themselves also can be the source of several plasma regimes, from hot plasma channels to coronas, frequently found simultaneously in them. Early observations (Schonland et al., 1935) highlighted the stepped propagation of downward negative leaders to ground. Streamers normally start at high-frequency rates from leader tips. The streamer zone at the leader tip ( $t_1$  in Fig. 4b) provides the charge that will surround—cover—the hot leader channel when the leader advances (Bazelyan and Raizer, 2000).

For instance, let us take EM pulses of order  $10^5$  V/m emitted in the 50 – 60 MHz band which is the typical band of negative leaders. The plasma density that the pulse encounters should be of the order  $10^{10}$  m $^{-3}$  in order to accelerate electrons to MeV energies under collisionless conditions. For a characteristic temperature of a plasma created in lightning (30 000 K), the characteristic plasma length  $\lambda_{De}$  should be the order of tens of centimeters.

Then the question is whether a plasma of size larger than 10 cm and electron density less than  $10^{10}$  m $^{-3}$  can be found in the vicinity of a stepped leader. If the air is fully ionized, the densities are too large at altitudes of 10–15 km and the mechanism would not work at such

altitudes. The particular plasma densities required in this discussion has been found in numerical simulations for sprite streamers in the Earth atmosphere (ParraRojas et al., 2013) and might be present in other transient events out of equilibrium at lower temperatures, but note that sprite halos occur in the Earth's upper atmosphere at altitudes of  $\approx 80$  km so the encounter with the halo of a pulse created at low altitudes able to trigger the wakefield acceleration would be unlikely.

## 5. Conclusions

In this paper we have studied the conditions for the creation of MeV electrons in atmospheric plasmas under wakefield acceleration in the collisionless case. An intense electromagnetic pulse interacting with the plasma can create a wake on the plasma. Electrons trapped in such oscillations can be accelerated under certain conditions to high energies. We have shown that those electrons could reach energies in the MeV range, thus being able to ignite gamma bursts. We have checked these results using numerical simulations.

The needed pulses and plasmas can be generated by electromagnetic activity in the atmosphere. We have discussed whether some particular case for the wakefield acceleration can be found in the Earth atmosphere and found some constrains.

In the case that the collisions could not be neglected, still the amplification of the pulse can be achieved via the coupling of the electromagnetic wave and the electron plasma by an ion density fluctuation (Kruer, 2003). In this case, conditions (3) and (4) would not hold and condition (5) should be modified. A full study of the ponderomotive force acting on the plasma is then needed and will be the subject of future work.

Our mechanism is not incompatible with other proposed mechanism for the creation MeV electrons.

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## Appendix A

Let us recall how an electromagnetic wave interacts with a plasma and how a plasma modifies the propagation of the electromagnetic waves (Kruer, 2003). The starting point to describe the evolution of a plasma is the Vlasov equation, which up to first order in the expansion parameter  $1/N_D$ , reads

$$\frac{\partial f_j}{\partial t} + \mathbf{v} \cdot \frac{\partial f_j}{\partial \mathbf{r}} + \frac{q_j}{m_j} (\mathbf{E} + \mathbf{v}_j \times \mathbf{B}) \cdot \frac{\partial f_j}{\partial \mathbf{r}} = \sum_k \left( \frac{\partial f_{jk}}{\partial t} \right)_c, \quad (\text{A.1})$$

being  $f_j(\mathbf{r}, \mathbf{v}, t)$  the distribution function of the  $j$  species. The right hand side of (A.1) represents the collisional terms. The parameter  $N_D$  is the number of particles in the Debye sphere. For electrons,  $N_D = 4\pi n_e \lambda_{De}^3 / 3$ , where  $n_e$  is the electron density and  $\lambda_{De}$  the electron Debye length. A useful expression of

the Debye length under equilibrium conditions is given by  $\lambda_{De} = 69\sqrt{T/n_e}$  (m), where  $T$  is the equilibrium temperature of the electrons in Kelvin and  $n_e$  is expressed in  $\text{m}^{-3}$  (Spitzer, 1962), so we have the condition

$$N_D \approx 1.38 \times 10^6 T^{3/2}/n_e^{1/2} \gg 1, \quad (\text{A.2})$$

for (A.1) to be valid. Note as pointed in the main text, that the condition to have a plasma is  $\lambda_{De} \ll L$ , being  $L$  the characteristic dimension of the system.

From (A.1) it is a standard procedure to calculate the first moment equations. We will assume that collisions do not change the number of species, so in the averaging we will take

$$\int d\mathbf{v} \sum_k \left( \frac{\partial f_{jk}}{\partial t} \right)_c = 0.$$

Hence, the change of the momentum of the  $j$  species becomes

$$n_j \frac{\partial \mathbf{u}_j}{\partial t} + n_j \mathbf{u}_j \cdot \frac{\partial \mathbf{u}_j}{\partial \mathbf{r}} = \frac{q_j}{m_j} n_j (\mathbf{E} + \mathbf{u}_j \times \mathbf{B}) - \frac{1}{m_j} \frac{\partial p_j}{\partial \mathbf{r}} - \sum_{k \neq j} \left( \frac{\partial}{\partial t} n_j \mathbf{u}_j \right)_k, \quad (\text{A.3})$$

where  $n_j$  is the density and  $\mathbf{u}_j$  the velocity of the  $j$  species and  $p_j$  the pressure.

Let us consider a plasma composed of ions with positive charged  $+eZ$  and electrons. We will assumed that ions form a fixed background with density  $n_i = n_e/Z$ , where  $n_e$  is the corresponding electron density. Then we only need to treat the dynamics of the electron fluid. We can investigate the damping of an electromagnetic wave of the form  $\mathbf{E}(\mathbf{r})\exp(-i\omega t)$  considering the linearised plasma response. Writing

$$\left( \frac{\partial}{\partial t} n_e \mathbf{u}_e \right)_i = \nu_{ei} n_e \mathbf{u}_e,$$

where  $\nu_{ei}$  is the collisional frequency of the scattering of electrons by ions, from (A.3) to first order we get

$$\frac{\partial \mathbf{u}_e}{\partial t} = -\frac{e}{m_e} \mathbf{E}(\mathbf{r}) e^{-i\omega t} - \nu_{ei} \mathbf{u}_e. \quad (\text{A.4})$$

The solution of (A.4) results

$$\mathbf{u}_e(\mathbf{r}, t) = \frac{-ie}{m_e(\omega + i\nu_{ei})} \mathbf{E}(\mathbf{r}) e^{-i\omega t}. \quad (\text{A.5})$$

The plasma conductivity  $\sigma$  can be calculated from the current density of the plasma  $\mathbf{j} = -en_e \mathbf{u}_e$ . Using (A.5)

$$\mathbf{j} = i\varepsilon_0 \frac{\omega_{pe}^2}{\omega + i\nu_{ei}} \mathbf{E}(\mathbf{r}) e^{-i\omega t} = \sigma \mathbf{E}(\mathbf{r}, t), \quad (\text{A.6})$$

where the plasma frequency is defined as

$$\omega_{pe}^2 = n_e e^2 / (\varepsilon_0 m_e), \quad (\text{A.7})$$

and the plasma conductivity  $\sigma$  is a complex quantity,

$$\sigma = i\varepsilon_0 \frac{\omega_{pe}^2}{\omega + i\nu_{ei}}. \quad (\text{A.8})$$

From Maxwell's equations for harmonic fields,

$$\nabla \times \mathbf{E}(\mathbf{r}) = i\omega \mathbf{B}(\mathbf{r}),$$

$$\nabla \times \mathbf{B}(\mathbf{r}) = \mu_0 \sigma \mathbf{E} - i \frac{\omega}{c^2} \mathbf{E}(\mathbf{r}). \quad (\text{A.9})$$

The second equation in (A.9) can be written as

$$\nabla \times \mathbf{B}(\mathbf{r}) = -i \frac{\omega}{c^2} \varepsilon \mathbf{E}(\mathbf{r}), \quad (\text{A.10})$$

where

$$\varepsilon = 1 - \frac{\omega_{pe}^2}{\omega(\omega + i\nu_{ei})}, \quad (\text{A.11})$$

is the dielectric function of the plasma. Taking the curl in equations (A.9) and (A.10), one gets the wave equations

$$\nabla^2 \mathbf{E}(\mathbf{r}) - \nabla(\nabla \cdot \mathbf{E}(\mathbf{r})) + \frac{\omega^2}{c^2} \varepsilon \mathbf{E}(\mathbf{r}) = 0,$$

$$\nabla^2 \mathbf{B}(\mathbf{r}) + \frac{1}{\varepsilon} \nabla \varepsilon \times (\nabla \times \mathbf{B}(\mathbf{r})) + \frac{\omega^2}{c^2} \varepsilon \mathbf{B}(\mathbf{r}) = 0, \quad (\text{A.12})$$

that give the spatial behaviour of the electric and magnetic fields in the plasma.

In the case of a neutral plasma with a uniform density  $\nabla \varepsilon = 0$  and  $\nabla \cdot \mathbf{E} = 0$ , for harmonic electromagnetic waves  $\mathbf{E}(\mathbf{r}) \sim \exp(i\mathbf{k} \cdot \mathbf{r})$ , equation (A.12) yield  $\omega^2 \varepsilon = c^2 k^2$ . So using (A.11)

$$\omega^2 = \omega_{pe}^2 \left( 1 - i \frac{\nu_{ei}}{\omega} \right) + c^2 k^2, \quad (\text{A.13})$$

in which it is assumed that  $\nu_{ei} \ll \omega$ . Equation (A.13) means that electromagnetic waves are damped. Writing  $\omega = \omega_r - i\nu/2$ , where  $\nu$  is the damping rate, equation (A.13) becomes

$$\omega_r = \sqrt{\omega_{pe}^2 + k^2 c^2},$$

$$\nu = \frac{\omega_{pe}^2}{\omega_r^2} \nu_{ei} \quad (\text{A.14})$$

The damping rate can be computed from the zero-order distribution function. This allows to get an expression for the collision frequency, namely (Kruer, 2003; Spitzer, 1962)

$$\nu_{ei} = \frac{1}{3(2\pi)^{3/2}} \frac{Z\omega_{pe}^4}{n_e \nu_e^3} \ln \Lambda = 3.61 \times 10^{-6} Z \ln \Lambda \frac{n_e}{T^{3/2}}, \quad (\text{A.15})$$

where we use  $\nu_e = \sqrt{K_B T / m_e}$  for the thermal velocity of electrons. The factor  $\Lambda$  is the ratio of the maximum and minimum impact parameters. The maximum impact parameter  $r_{max}$  is given by the Debye length  $\lambda_{De}$ , as the Coulomb potential is shielded out over that distance. The minimum impact parameter is given by the classical distance of closest approach  $r_{min} = Ze^2 / 4\pi\epsilon_0 m_e \nu_e^2$ , which averaging over all the particle velocities and assuming a Maxwellian distribution yields  $\bar{r}_{min} = Ze^2 / (12\pi\epsilon_0 K_B T)$ , so

$$\Lambda = \frac{\lambda_{De}}{\bar{r}_{min}} = \frac{12\pi n_e \lambda_{De}^3}{Z} \quad (\text{A.16})$$

For typical values of plasmas,  $\ln \Lambda \approx 10$ . We must notice that (A.15) is only an approximation where the zero order electron distribution is taken Maxwellian. For non equilibrium processes, the collisional damping rate could be less. For example in the case of a super-Gaussian distribution, the collisional damping is reduced by a factor of 2.

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