# Strong enhancement of noise-induced escape by nonadiabatic periodic driving due to transient chaos

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We have found a mechanism by which a moderately weak *nonadiabatic* periodic driving may significantly facilitate noise-induced interwell transitions in an *underdamped* multiwell system. The mechanism is associated with the onset of a *homoclinic tangle* in the noise-free system: if the ratio of the driving amplitude $A$ to the damping $\Gamma$ exceeds a critical value $\sim 1$, then the basins of attraction of the linear responses related to different wells are mixed in a complex manner in some layer associated with the separatrix of the undriven nondissipative system, and the minimal energy in such layer is lower than the top of the barrier. Thus the energy to which the system needs to be activated by the noise, to be able to make a transition, is lower than the top of the barrier.

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The influence of weak *nonadiabatic* periodic driving on noise-induced escape is a fundamental problem whose solution is far from completion, despite numerous studies (e.g., Ref. [1–6]). It is also relevant to many applications, e.g., to the destruction of metastable states in devices based on Josephson junctions [1,4] or in mechanical electrometers [7], and to directed diffusion [5,6].

Unlike most of the works on stochastic resonance (see Refs. [8,9] for reviews) and early works on directed diffusion (e.g., Ref. [10]), which relate to *adiabatic* driving (when the escape rates are determined by the instantaneous value of the driving force), the escape rate for *nonadiabatic* driving does not manage to follow changes of the driving force. If the temperature is not too small, the main effect of the driving was shown [3] to be an enhancement of the diffusion over the energy, which increases only the *prefactor* in the escape rate (still weakly: the correction is quadratic in the driving amplitude, which is small). But if the temperature is smaller than the driving amplitude, the effect of nonadiabatic driving was recently shown [5] to be much stronger: the mechanism in Ref. [5] was based on positive work by the force, pushing the system resonantly with the eigenoscillation at the resonant energy, thus freeing the noise from this work in the range of energies close to the resonant one, which lowers the *activation energy*.

However, the increase of the escape rate predicted in Refs. [3,5] *diverges* as the damping $\Gamma$ goes to zero. On the one hand, this suggests that the system should be *underdamped* in order for the increase to be *maximally* possible. On the other hand, neither of these theories can determine what are (i) the maximum increase, (ii) the proper conditions for it, and (iii) the underlying mechanism. Thus it is extremely important to study the problem in the underdamped limit, which is the major purpose of this paper.

As an example of systems possessing a barrier, we consider the double-well Duffing oscillator (Fig. 1):

$$\Pi(\ddot{q},\dot{q},q,t)=\xi(t), \quad \Pi=\ddot{q}+2\Gamma\dot{q}+U'(q)-A\cos(\Omega t), \tag{1}$$

$$\langle\xi(t)\xi(t')\rangle=4\Gamma T\delta(t-t'), \quad U(q)=-q^2/2+q^4/4.$$

Using the concept of *large fluctuations* (see, e.g., Refs. [11,12,5]; the full variety of modifications of the concept is reviewed in [13,14]), one can show that the transition rate $W$ between steady regimes $q_{st}^{(1,2)}(t)$ of the forced vibrations around the minima of the potential $U(q)$ can be described by an activation law $W\propto\exp(-S_a/T)$, where the activation energy $S_a$ is some functional minimized over the end state $\vec{s}_e$ [any state in the phase space from which the noise-free system can relax both into $q_{st}^{(2)}(t)$ and $q_{st}^{(1)}(t)$], over the end time $t_e$ and over the path $[q(t)]\equiv[q_{st}^{(1)}\overset{t_e}{\rightarrow}\vec{s}_e]$:

$$S_a=\min_{[q(t)],\vec{s}_e,t_e}S, \quad S=\frac{1}{8\Gamma}\int_{-\infty}^{t_e}dt\Pi^2(\ddot{q},\dot{q},q,t), \tag{2}$$

$$q(t\rightarrow-\infty)\rightarrow q_{st}^{(1)}(t), \ \{q(t_e),\dot{q}(t_e)\}=\vec{s}_e\overset{nf}{\rightarrow}q_{st}^{(1,2)}(t),$$

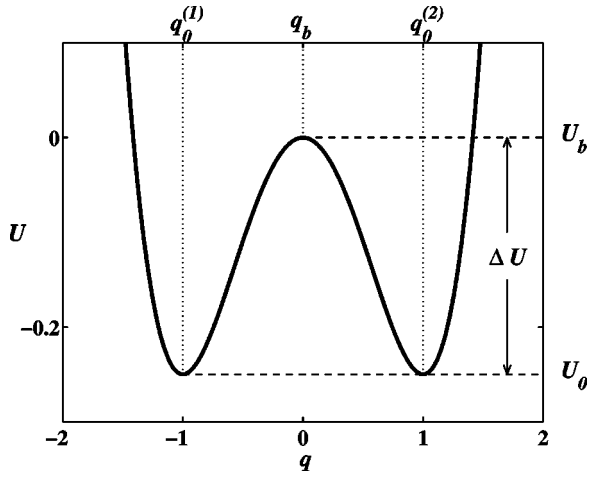


FIG. 1. Duffing potential $U(q)=-q^2/2+q^4/4$.

 

$$q_{st}^{(1,2)}(t)\approx q_0^{(1,2)}+\frac{A}{2-\Omega^2}\cos(\Omega t),\quad \Gamma\ll|\Omega-\sqrt{2}|.$$

If $A=0$, then all states $\{q,\dot{q}\}$ corresponding to $q_{st}^{(1,2)}(t)$ reduce to the *stationary* stable states $\vec{s}_{1,2}$ of the undriven system while the *exit state* $\vec{s}_{ex}$, i.e., $\vec{s}_e$ minimizing $S$, reduces to the *unstable* stationary state $\vec{u}$ at the top of the barrier (the exit time $t_{ex}$, i.e., $t_e$ minimizing $S$, reduces to $\infty$). The path $[q(t)]$ yielding $S_a$ for the transition $\vec{s}_1\to\vec{u}$, called the most probable escape path ($\mathcal{P}_{\rm MPEP}$), is the time reversal of the noise-free trajectory $[\vec{u}\overset{nf}{\to}\vec{s}_1]$ (see, e.g., Refs. [11,5]):

$$\mathcal{P}_{\rm MPEP}(A=0)\equiv[Q(t)],\quad \ddot{Q}-2\Gamma\dot{Q}+U'(Q)=0,$$
$$Q(-\infty)=q_0^{(1)},\quad \dot{Q}(-\infty)=0,\quad Q(\infty)=q_b,\quad \dot{Q}(\infty)=0. \tag{3}$$

The path $[Q(t)]$ provides $S_a=\Delta U\equiv U_b-U_0$, which obviously agrees with the classical result [15].

The presence of the periodic driving affects $S_a$ in three different ways: (1) a shift of the exit energy $E_{ex}\equiv E(\vec{s}_{ex})\equiv\dot{q}_{ex}^2/2+U(q_{ex})$ from $E(\vec{u})=U_b$; (2) a shift of the starting energy $E_{st}$, i.e., energy of the starting state $\vec{s}_{st}$ (belonging to $q_{st}^{(1)}$), from $E(\vec{s}_1)=U_0$; and (3) a breakdown of the relation $S_a=E_{ex}-E_{st}$. Correspondingly, we formally divide $\delta S_a\equiv S_a-\Delta U$ into three parts:

$$\delta S_a\equiv\delta S_a^{(ex)}+\delta S_a^{(st)}+\delta S_a^{(r)},\quad \delta S_a^{(ex)}\equiv E_{ex}-U_b,$$
$$\delta S_a^{(st)}\equiv U_0-E_{st},\quad \delta S_a^{(r)}\equiv S_a-(E_{ex}-E_{st}). \tag{4}$$

Before considering these separately in items (1)–(3) below, we need to briefly review the relevant results of Ref. [5].

In the asymptotic limit $A\to 0$, the leading-order correction to the $\mathcal{P}_{\rm MPEP}$ $(A=0)\equiv[Q(t)]$ is linear in $A$ [5] (in particular, this concerns $\vec{s}_{st}$ and $\vec{s}_{ex}$). As follows from the definition of the $\mathcal{P}_{\rm MPEP}$ [11,5], corrections to $S_a(A=0)\equiv\Delta U$ from a linear correction of $\mathcal{P}_{\rm MPEP}$ are weaker than linear. Hence, to calculate the leading-order (linear) term in $\delta S_a$, one may use $[Q(t)]$ [5]:

$$\delta S_a\approx\delta S_a^{(r)}\approx-|\chi|A,\quad \chi=-\int_{-\infty}^{\infty}dt e^{i\Omega t}\dot{Q}(t). \tag{5}$$

If $\Gamma\ll 1,\Omega$, the most important contributions to $\chi(\Omega)$ are provided by those bits of $[Q(t)]$ which correspond to energies $E\equiv\dot{Q}^2/2+U(Q)$ close to the *resonant* energies $E_n(\Omega)$, implicitly defined as $n\omega(E_n)=\Omega$, where $\omega(E)$ is the frequency of eigenoscillation with energy $E$, and $n$ is an integer. Labeling with $N$ the resonance which provides the largest contribution to $\delta S_a$, one obtains [5]

$$|\chi|\approx\chi_N\equiv|\dot{Q}_N(E_N)|\int_{-\infty}^{\infty}dt\cos\left(N\frac{d\omega(E_N)}{dE_N}2\Gamma\omega(E_N)I(E_N)t^2\right)$$
$$=\left[|\dot{Q}_N|\left(2\Gamma NI\left|\frac{d\omega^2}{dE}\right|\Big/\pi\right)^{-1/2}\right]_{E=E_N}\propto\frac{1}{\sqrt{\Gamma}}\overset{\Gamma\to 0}{\to}\infty \tag{6}$$

[here $\dot{Q}_N(E_N)$, $\omega(E_N)$ and $I(E_N)$ are the amplitude of the $N$th overtone of the velocity, frequency and action for the eigenoscillation at energy $E_N$, respectively]. It is taken into account in Eq. (6) that, along $Q(t)$, $\dot{E}\approx 2\Gamma\omega(E)I(E)$, while $\omega(E)\approx\omega(E_N)+d\omega(E_N)/dE_N(E-E_N)$, and $t=0$ is defined as $E(t=0)=E_N$.

The divergence of $\chi$ may give the impression that, as $\Gamma$ is decreased to $\sim A^2$, the escape probability grows to $\sim 1$ [16]. But, as we will see, this impression is wrong: Eq. (5) is invalid for $\Gamma\lesssim A$, and the contribution from the resonant mechanism vanishes rather than grows as $\Gamma\to 0$, while $\delta S_a$ is dominated by $\delta S_a^{(ex)}$ at $\Gamma\ll A^2$.

(1) To evaluate $\delta S_a^{(ex)}$ we note first that $\delta S_a^{(r)}$ is dominated by the resonant mechanism [5], which, for $\Gamma\ll 1$, does not involve $\vec{s}_e$ [cf. Eqs. (5) and (6), as well as item (3) below]. Therefore, $\delta S_a^{(ex)}$ can be derived from the minimization of $E(\vec{s}_e)$ over $\vec{s}_e$, independently of $\delta S_a^{(r)}$ and $\delta S_a^{(st)}$. So, to leading order in $A$, $\vec{s}_{ex}$ is the state which, among all possible $\vec{s}_e$, has the minimal energy $E_m$:

$$\delta S_a^{(ex)}\approx E_m-U_b. \tag{7}$$

For $A\to 0$, $\vec{s}_{ex}$ belongs to the unstable periodic orbit near the top of the barrier [5]. So $U_b-E_m\approx A^2/[2(1+\Omega^2)^2]$ and $-\delta S_a^{(ex)}\ll-\delta S_a^{(r)}\propto A$ can be neglected.

On the other hand, if

$$A>A_c\approx\mu_l\Gamma,\quad \mu_l=\frac{4\sqrt{2}\cosh(\pi\Omega/2)}{3\pi\Omega}, \tag{8}$$

($\mu_l\sim 1$ at $\Omega\sim 1$, so that $A_c\sim\Gamma$), then a *homoclinic tangle* arises in the *noise-free* system [17], leading, in the Poincaré section, to a complex mixing of the basins of attraction of $q_{st}^{(1,2)}$ in a layer around the boundary between the basins of attraction of the stable states of the undriven system (Fig. 2). To first order in $A$, $E_m$ is the minimum energy in that part of the basin of attraction of $q_{st}^{(2)}$ where $q<0$, additionally minimized over the angle of the Poincaré section. It can be shown (cf. Ref. [17]) that, to first order in $A$, $E_m<U_b$ if and only if condition (8) holds. If Eq. (8) holds and $\Omega\sim 1$, then $(U_b-E_m)/A\sim 1$.

One can rather easily find $E_m$ numerically, merely integrating the dissipative equation (1) in the absence of noise $(T=0)$ on a grid of initial states with $q<0$, and choosing from them the state which has the minimal energy among all the states which provide a relaxation to the attractor $q_{st}^{(2)}$: this energy approximates $E_m$ to first order in $A$.

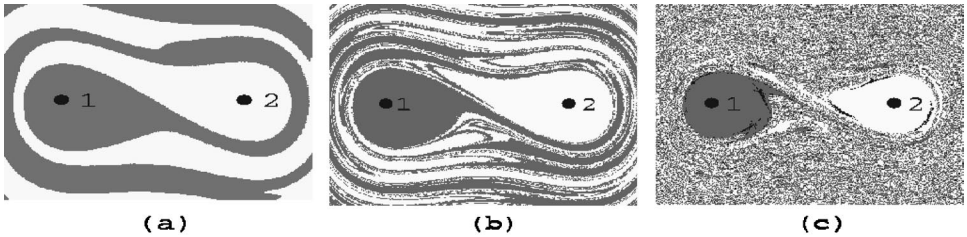


FIG. 2. Stroboscopic ($\Omega t = 0, 2\pi, 4\pi, \ldots$) Poincaré section $\dot{q} - q$ of the noise-free ($T=0$) system (1) for $A=0.07$, $\Omega=1.7$ while $\Gamma$ decreases: (a) $\Gamma=0.07$, (b) $\Gamma=0.025$, and (c) $\Gamma=0.005$. Attractors corresponding to $q_{st}^{(1,2)}$ are marked by dots, and label 1 and 2. Their basins of attraction are shown by different shades of grey [small black areas in (c) are basins of attraction of period-3 orbits]. The mixing of basins is (a) absent, (b) already present, and (c) well developed.

Moreover, for $A \gg A_c \sim \Gamma$, the numerical search for $E_m$ can be additionally simplified significantly: the lower-energy boundary of the layer then coincides with the lower-energy boundary of the corresponding chaotic layer of the *nondissipative* system (namely, of the chaotic layer which includes the state $\{q=q_b,\ \dot{q}=0\}$), while it can be shown that the minimal energy in a Poincaré section of the chaotic layer, $E_m^{(nd)}$, is independent of the section angle; thus, $E_m \approx E_m^{(nd)}$. The explicit formula for $E_m^{(nd)}$ is not known [18], but the chaotic layer is readily generated by computer, so that $E_m^{(nd)}$ can be very easily found numerically. Its detailed analysis will be presented elsewhere; here we present characteristic examples of the dependence of $U_b - E_m^{(nd)}$ on the amplitude and frequency of the driving force. The amplitude dependence is stairlike [Fig. 3(a)], while the frequency dependence has sharp peaks [Fig. 3(b)]. Jumps in the former dependence and peaks in the latter correspond to the overlap and separation between nonlinear resonances. Let us first demonstrate this for the frequency dependence.

For very small $\Omega$, the relevant chaotic layer relates only to the separatrix of the undriven system, and $U_b - E_m^{(nd)} \propto \Omega A$ [19]. As $\Omega$ grows, the resonant energy $E_1(\Omega)$ sharply lowers and, starting from $\Omega = \Omega_1 \approx 2\pi/\ln(1/A)$, the lower part of the chaotic layer relates to the lower part of the *nonlinear resonance* [14–16] while the upper part of the layer still relates to the separatrix of the *undriven* system [both parts are clearly resolved in the Poincaré section: Fig. 4(a)]. Thus $U_b - E_m^{(nd)}$ grow sharply, and reaches the first maximum for $\Omega$ slightly larger than $\Omega_1$ while, as $\Omega$ grows further, the layer related to the nonlinear resonance separates from the layer around the original separatrix and, thus, can no longer provide the interwell chaotic transport [Fig. 4(b)]: due to this, $U_b - E_m^{(nd)}$ drops abruptly. Peaks at the multiple frequencies correspond to higher-order resonances.

Similarly, as $A$ grows, $U_b - E_m^{(nd)}$ undergoes large jumps at $A_n \sim \exp(-2\pi n/\Omega)$, related to successive overlaps between the original layer and the layers associated with nonlinear resonances. Note, however, that the largest $A_n$, namely, $A_{[\Omega/\sqrt{2}]+1}$, is still typically quite small (unless $\Omega$ is only slightly less than the eigenfrequency at the bottom, $\sqrt{2}$) and the further growth of $U_b - E_m^{(nd)}$ with $A$ is approximately linear. Thus, for most of the range $A \ll 1$, the quantity $(U_b - E_m^{(nd)})/A$ plays a role similar to the role of $|\chi|$ in Eq. (5).

(2) The maximum deviation of energy on the attractor $q_{st}^{(1)}(t)$ from $U_0$ is $\approx \{A \max(\Omega, \sqrt{2})/(2-\Omega^2)\}^2/2$. Thus

$$-\delta S_a^{(st)} \equiv E_{st} - U_0 < \frac{1}{2}\left(\frac{\max(\Omega,\sqrt{2})}{2-\Omega^2}\right)^2 A^2, \qquad (9)$$

and may be neglected in comparison with $-\delta S_a^{(ex)}$ if

$$A \ll A_m \equiv 8(1-\Omega/\sqrt{2})^2, \quad \Omega \sim 1. \qquad (10)$$

Unless $\Omega$ is very close to $\sqrt{2}$, $A_m \lesssim 1$, so that $\delta S_a^{(st)}$ may be neglected in the range under study, $A \ll 1$.

(3) To leading (linear) order in $A$, the correction $\delta S_a^{(r)}$ in the presence of the layer may be described analogously to when in the absence of the layer [cf. Eq. (5)],

$$\delta S_a^{(r)} \approx -|\tilde{\chi}|A, \quad \tilde{\chi} = -\int_{t_{st}}^{t_{ex}} dt e^{i\Omega t} \ddot{\tilde{Q}}(t), \qquad (11)$$

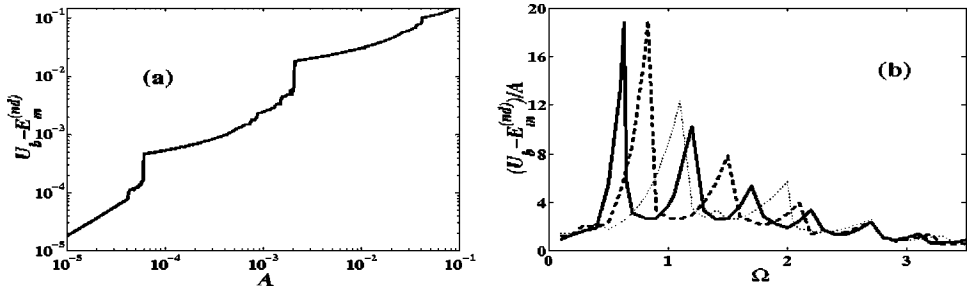


FIG. 3. (a) $U_b - E_m^{(nd)}$ as a function of $A$ (note logarithmic scales) for $\Omega=1.7$. (b) $(U_b - E_m^{(nd)})/A$ as a function of $\Omega$ for $A=0.0001$, 0.001, and 0.01 (solid, dashed, and dotted lines, respectively). $E_m^{(nd)}$ is the minimal energy in the chaotic layer in the Poincaré section of the *nondissipative* system (cf. Fig. 4).

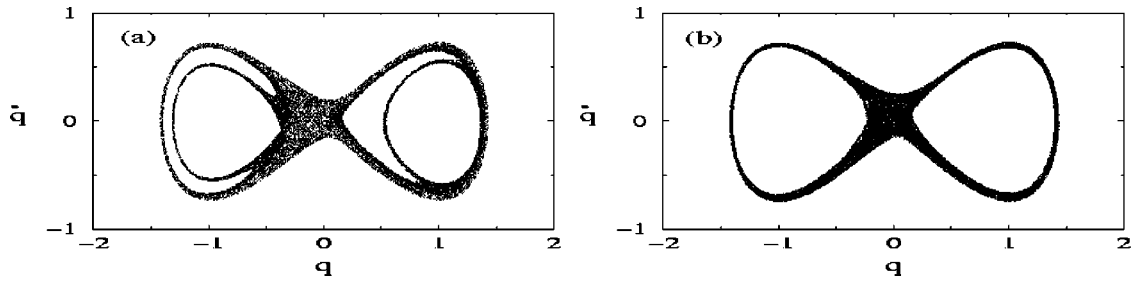


FIG. 4. The chaotic layer (black) which provides the *interwell* chaotic transport in the nondissipative noise-free system, for $A=0.01$. (a) $\Omega=1.1$, and (b) $\Omega=1.2$.

where $\tilde{Q}(t)$ is the time reversal of the noise-free relaxation from $\vec{s}_{ex}$ to $\vec{s}_{st}\equiv\{q(t_{st}),\dot{q}(t_{st})\}$. As in the absence of the layer, the main mechanism contributing to $\tilde{\chi}$ at small $\Gamma$ is a resonant one, so that $|\tilde{\chi}|\approx\chi_N$ as in Eq. (6).

Let us first estimate the range of $\Gamma$ at which the resonant mechanism saturates and Eq. (11) is no longer valid. The main contribution to the integral in Eq. (6) comes from the range of $t$ during which the absolute value of the argument of the cosine is $\lesssim\pi/2$, i.e., $|t|\lesssim t_r \equiv\sqrt{\pi/[4\Omega|d\omega(E_N)/dE_N|\Gamma I(E_N)]}\propto\Gamma^{-1/2}$. In this range, the energy along the $\mathcal{P}_{\mathrm{MPEP}}$, $E\approx\dot{\tilde{Q}}^2/2+U(\tilde{Q})$, increases from $E_N-\Delta E_r/2$ to $E_N+\Delta E_r/2$, where

$$\Delta E_r\approx 2\Gamma\omega(E_N)I(E_N)2t_r\propto\sqrt{\Gamma}. \tag{12}$$

The ''unperturbed'' part of the activation energy associated with a noise-induced increase of energy for $\Delta E_r$ is equal to $\Delta E_r$. Thus the perturbative formula [Eq. (11)] is valid as long as the absolute value of the negative correction by the resonant mechanism in the range $[-t_r,t_r]$, which is $\sim|\dot{Q}_N(E_N)|t_rA$, is less than $\Delta E_r$. Hence the range of the validity of Eq. (11) is

$$A\ll\mu_r\Gamma,\quad \mu_r=[\omega I/|\dot{Q}_N|]_{E=E_N}. \tag{13}$$

Typically, $\mu_r\gtrsim 1$. So, as $\Gamma$ decreases, the growth of $-\delta S_a$ due to the resonant mechanism saturates at $\Gamma\sim A$.

The next question is what is $\delta S_a^{(r)}$ for $\Gamma\ll A$? We have not succeeded in a rigorous treatment of this problem, which is extremely difficult, but we suggest an intuitive argument in favor of a vanishing correction as $\Gamma\to 0$ (computer simulations confirm this; see below): the resonant mechanism affects mainly the ''resonant'' energies, i.e., those in the band $[E_N-\Delta E_r/2,E_N+\Delta E_r/2]$; hence the absolute value of the correction cannot significantly exceed the width of this band,

$$-\delta S_a^{(r)}\lesssim\Delta E_r\propto\sqrt{\Gamma}\overset{\Gamma\to 0}{\to}0,\quad \Gamma\lesssim A. \tag{14}$$

Comparing the contributions considered in items (1)–(3), we conclude that, provided

$$\sqrt{\Gamma}\ll A\ll 8(1-\Omega/\sqrt{2})^2,1, \tag{15}$$

the *layer mechanism* dominates in $\delta S_a$:

$$-\delta S_a\approx-\delta S_a^{(ex)}\approx U_b-E_m^{(nd)}\sim A. \tag{16}$$

Thus, if $A$ increases (while $\Gamma\ll 1$), $-\delta S_a$ evolves as follows: for $A\ll\Gamma$, it grows in accordance with Ref. [5], i.e., $-\delta S_a\approx-\delta S_a^{(r)}\approx|\chi|A$; for $A\sim\Gamma$, it saturates; for $A\gtrsim\Gamma$, it grows again, due to the lowering $E_{ex}$ [see Eqs. (7) and (16)].

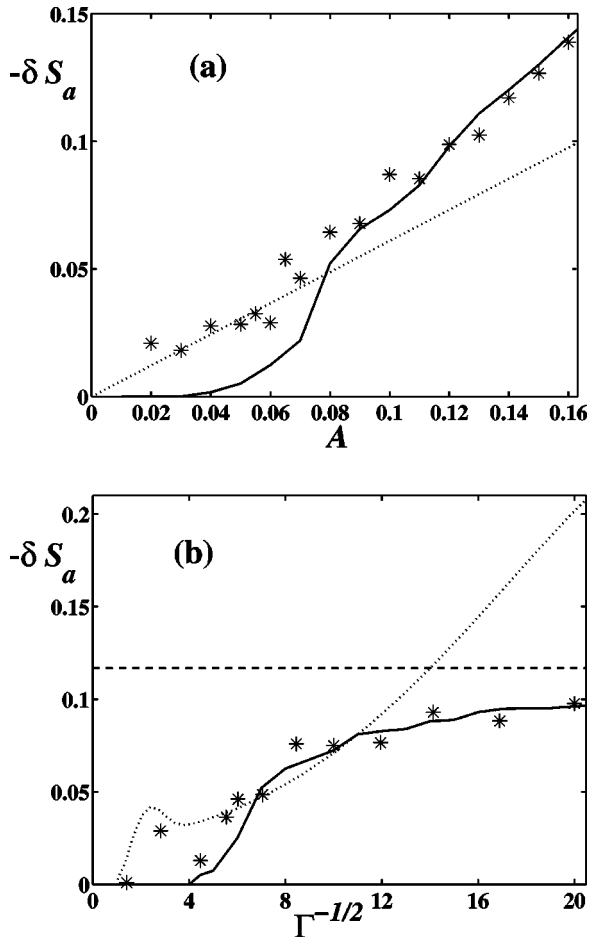


FIG. 5. $-\delta S_a$ as function of (a) $A$ (for $\Gamma=0.025$), or (b) $\Gamma^{-1/2}$ (for $A=0.07$); $\Omega=1.7$ in both cases. Stars are computer simulations of Eq. (1), using Eq. (17); dotted lines are theory for $-\delta S_a^{(r)}$, based on the resonant mechanism [5] [see Eq. (5)]; solid lines are theory, based on the layer mechanism [Eq. (7)], for $U_b-E_m$. The dashed line in (b) shows $U_b-E_m^{(nd)}$, which is our theoretical nondissipative asymptote both for $-\delta S_a$ and for the solid line [see Eq. (16)].

Note that, if $1 \gg A \gg \sqrt{\Gamma}, A_{[\Omega/\sqrt{2}]+1}$, then the growth is approximately linear [cf. Figs. 3(a) and 5(a)].

If, fixing $A \ll 1$, we decrease $\Gamma$, then $-\delta S_a$ evolves as follows: for $\Gamma \gg A$, it is given by Eq. (5), typically growing $\propto \Gamma^{-1/2}$ if $\Gamma \ll 1$; for $\Gamma \sim A$, it saturates at $\sim \Delta E_r \propto \sqrt{A}$, while reaching the asymptotic limit [Eq. (16)] for $\Gamma \ll A^2$. Thus, in the asymptotic limit $A \to 0$, the function $-\delta S_a(\Gamma)$ possesses a maximum ($\propto \sqrt{A}$) at $\Gamma \sim A$. At the same time, if $A$ is *moderately* small, then the maximum ($\propto A$) is reached at $\Gamma \to 0$ [cf. Fig. 5(b)].

To test our theoretical predictions, we numerically simulated Eq. (1), and measured the transition flux $J \equiv J(A,T)$ from $q_{st}^{(1)}(t)$ to $q_{st}^{(2)}(t)$ (at small temperatures). In order to reduce as much as possible the influence of the prefactor $P$ in the determination of $S_a$ [note that $J(A,T) = P(A,T)\exp(-S_a(A)/T)$], we simulated Eq. (1) for two slightly different temperatures and measured the flux both for a given $A$ and for $A=0$; an activation energy was then calculated as

$$S_a \approx \frac{T_1 T_2}{T_1 - T_2} \ln\left(\frac{\tilde{J}(A,T_1)}{\tilde{J}(A,T_2)}\right),$$

$$\tilde{J}(A,T) \equiv J(A,T)/J(A=0,T), \tag{17}$$

$$T_1 \ll \Delta U, \quad |T_1 - T_2| \sim T_1^2/\Delta U.$$

Figure 5 shows that the crossover between the ''resonant'' and ''layer'' mechanisms in $\delta S_a(A)$, $\delta S_a(\Gamma^{-1/2})$ occurs in good agreement with the theoretical predictions, and that Eq. (16) is well satisfied. Moreover, for the given parameters, the layer mechanism becomes dominant even before the layer approaches its nondissipative limit.

Let us briefly discuss the application of our results to the problem of directed diffusion in periodic potentials at low damping [5]. The theory [5] predicts that the activation energies for the escape to the adjacent well from the left and from the right are typically different (so that the fluxes to the left and to the right differ exponentially strongly); moreover, this difference grows $\propto \Gamma^{-1/2}$ as $\Gamma \to 0$. However, as follows from the results of the present paper, this growth saturates at $\Gamma \sim A$ (i.e., long before the correction [5] to the activation energy becomes comparable to the potential barrier, which occurs at $\Gamma \sim A^2$) and then vanishes, since a layer with mixed basins (transient chaos) is formed; as soon as the system reaches any point of this layer it may then be transported to the well from the left and to the well from the right, with probabilities of the same order [20].

Finally, we put our work into the context of studies of the interplay between chaos and noise (cf. Ref. [17]). Most such works studied the effect of noise on transport properties *within* a chaotic attractor or layer or web. In Ref. [22], the dependence on noise intensity for noise-induced interattractor hoppings in some multiattractor map with transient chaos was studied in simulations. But neither of these works studied how transient chaos (arising due to periodic driving) affects the noise-induced escape.

In conclusion, we have found the range of $\Gamma$ where the decrease of the lifetime by the nonadiabatic periodic driving is at a maximum: depending on parameters, it is either $\Gamma \to 0$ or $\Gamma \sim A$. In the former case, we provide a *quantitative* theory for the maximum decrease of the activation energy ($\sim A$) while, in the latter case, we estimate it qualitatively ($\sim \sqrt{A}$). The underlying mechanisms are (i) transient chaos if $\Gamma \to 0$, and (ii) a resonant (saturated) mechanism [5] if $\Gamma \sim A$.

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